



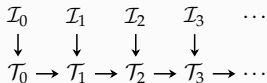
AXIOMATIZATION OF GENERAL CONCEPT INCLUSIONS FROM STREAMS OF INTERPRETATIONS WITH OPTIONAL ERROR TOLERANCE

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Introduction: Two Problems

Global Problem

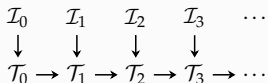


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Output: Sequence of TBoxes \mathcal{T}_n such that $\mathcal{T}_n \models C \sqsubseteq D$ if, and only if, $\mathcal{I}_1, \dots, \mathcal{I}_n \models C \sqsubseteq D$

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Local Problem

Input: A TBox \mathcal{T} and an interpretation \mathcal{I}

Output: A TBox \mathcal{U} such that $\mathcal{U} \models C \sqsubseteq D$ if, and only if, $\mathcal{T} \models C \sqsubseteq D$ as well as $\mathcal{I} \models C \sqsubseteq D$

The Description Logic \mathcal{EL}^\perp

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- *Terminological Box (TBox)* \mathcal{T} is a (finite) set of GCIs

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Semantics

- Interpretation $\mathcal{I} := (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ where $\Delta^{\mathcal{I}} \neq \emptyset$ is the domain and $\cdot^{\mathcal{I}}$ is a mapping such that $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ for all concept names $A \in N_C$ and $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ for all role names $r \in N_R$

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- $\mathcal{I} \models \mathcal{T}$ if $\mathcal{I} \models C \sqsubseteq D$ for all GCIs $C \sqsubseteq D \in \mathcal{T}$
- $\mathcal{T} \models C \sqsubseteq D$ if $\mathcal{I} \models C \sqsubseteq D$ for all models $\mathcal{I} \models \mathcal{T}$

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- $\mathcal{T} \models C \sqsubseteq D$ if $\mathcal{I} \models C \sqsubseteq D$ for all models $\mathcal{I} \models \mathcal{T}$
- $\mathcal{T} \models \mathcal{U}$ if $\mathcal{I} \models \mathcal{U}$ for all models $\mathcal{I} \models \mathcal{T}$

The Description Logic \mathcal{EL}^\perp

Least Common Subsumer

The *least common subsumer (lcs)* $C \vee D$ of \mathcal{EL}^\perp -concept descriptions C and D is defined by the following conditions:

- $\emptyset \models C \sqsubseteq C \vee D$
- $\emptyset \models D \sqsubseteq C \vee D$
- For all \mathcal{EL}^\perp -concept descriptions E , $\emptyset \models \{C \sqsubseteq E, D \sqsubseteq E\}$ implies $\emptyset \models C \vee D \sqsubseteq E$.

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The Lattice of \mathcal{EL}^\perp -Concept Descriptions

$\mathcal{EL}^\perp(N_C, N_R) := (\mathcal{EL}^\perp(N_C, N_R), \sqsubseteq) / \equiv$ is a lattice where

- \sqsubseteq denotes subsumption w.r.t. the empty TBox \emptyset
- \equiv is the induced equivalence relation of \sqsubseteq
- Conjunction \sqcap is the *infimum*
- Lcs \vee is the *supremum*
- \perp is the smallest, and \top the greatest element

An Existing Static Approach

- Baader and Distel have found a method for the construction of a base of GCIs for (finite) interpretations.
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Given: Interpretation \mathcal{I} , subset $X \subseteq \Delta^{\mathcal{I}}$

The *model-based most specific concept description (mmsc)* $X^{\mathcal{I}}$ of X with respect to \mathcal{I} is defined by the following conditions:

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- Existence of mmscs either via gfp-semantics or restriction of role depth.
- A slight reformulation of their results yields the following statements.
- The mapping $C \mapsto C^{\mathcal{I}\mathcal{I}}$ is a closure operator in the dual of $\mathcal{EL}^{\perp}(N_C, N_R)$.
- A base of $C \mapsto C^{\mathcal{I}\mathcal{I}}$ is a base of GCIs for \mathcal{I} .

Most Specific Consequences and a new Closure Operator

Most Specific Consequence

Given: \mathcal{EL}^\perp -concept description C , \mathcal{EL}^\perp -TBox \mathcal{T}

The *most specific consequence* $C^{\mathcal{T}}$ of C with respect to \mathcal{T} is defined by the following conditions:

$$\mathcal{T} \models C \sqsubseteq C^{\mathcal{T}}.$$

For all \mathcal{EL}^\perp -concept descriptions D , $\mathcal{T} \models C \sqsubseteq D$ implies $\mathcal{T} \models C^{\mathcal{T}} \sqsubseteq D$.

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Observation (Solution for the Local Problem)

To construct a base of GCIs that are both valid in \mathcal{I} and entailed by \mathcal{T} , it suffices to compute a base of implications of the infimum of $C \mapsto C^{\mathcal{II}}$ and $C \mapsto C^{\mathcal{T}}$.

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A base of the supremum of $C \mapsto C^{\mathcal{I}}$ and $C \mapsto C^{\mathcal{T}}$ yields a base for only GCIs that are valid for the individuals in \mathcal{I} which respect the GCIs in \mathcal{T} , i.e., satisfy some constraints (e.g. hand-crafted by an expert).

Error-Tolerance

- Dualizing the previous approach, we get an error-tolerant way for axiomatizing GCLs from an interpretation.

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A base of the supremum of $C \mapsto C^{\mathcal{I}}$ and $C \mapsto C^{\mathcal{T}}$ yields a base for only GCLs that are valid for the individuals in \mathcal{I} which respect the GCLs in \mathcal{T} , i.e., satisfy some constraints (e.g. hand-crafted by an expert).

- It is also possible to combine both approaches (infima/suprema) to construct bases of GCLs from streams of possibly erroneous interpretations.

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Questions?