

Biclustering Based on FCA and Partition Pattern Structures for Recommendation Systems

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- 1 Introduction
- 2 Biclustering
- 3 CC Biclustering
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 - Recommendation
- 4 CEC Biclustering
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- 5 Conclusion



- An European project who support the emergence of a European **cultural heritage** by allowing visitors in different cultural sites to **improve the quality of their visit** by using adapted **computer-based devices** and to consider the visit at a European level.



- An European project who support the emergence of a European **cultural heritage** by allowing visitors in different cultural sites to **improve the quality of their visit** by using adapted **computer-based devices** and to consider the visit at a European level.
- Such improvement can be accomplished by studying a **dynamic recommendation system**.

- Collaborative recommendations: studying **previous users** who have similar interest to a target user.

Objective

- Collaborative recommendations: studying **previous users** who have similar interest to a target user.
- Using **biclustering** to retrieve similar users.

Objective

- Collaborative recommendations: studying **previous users** who have similar interest to a target user.
- Using **biclustering** to retrieve similar users.
- Using **partition pattern structures** to obtain biclusters.

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Simultaneous clustering of both rows and columns of a data matrix.

A matrix

4	2	5	3	3
4	2	5	3	3
4	3	5	2	2
2	1	3	9	7
8	3	9	8	6

Simultaneous clustering of both rows and columns of a data matrix.

A constant-value bicluster

4	2	5	3	3
4	2	5	3	3
4	3	5	2	2
2	1	3	9	7
8	3	9	8	6

A constant-columns (CC) bicluster

4	2	5	3	3
4	2	5	3	3
4	3	5	2	2
2	1	3	9	7
8	3	9	8	6

A constant-rows bicluster

4	2	5	3	3
4	2	5	3	3
4	3	5	2	2
2	1	3	9	7
8	3	9	8	6

A coherent-evolution-on-columns (CEC) bicluster

4	2	5	3	3
4	2	5	3	3
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A coherent-evolution-on-rows bicluster

4	2	5	3	3
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Definition

A partition $\mathfrak{d} = \{p_i\}$ of a set G is a collection of $p_i \subseteq G$ such that:

$$\bigcup_{p_i \in \mathfrak{d}} p_i = G \quad \text{and} \quad p_i \cap p_j = \emptyset \quad \text{whenever} \quad i \neq j.$$

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Given $G =$ set of objects, and $M =$ set of attributes, $\delta : M \rightarrow D$.

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Given $G =$ set of objects, and $M =$ set of attributes, $\delta : M \rightarrow D$.

Example

	m_1	m_2	m_3	m_4	m_5
g_1	1	5	3	4	7
g_2	1	1	4	2	7
g_3	2	5	4	5	3
g_4	2	5	4	5	7

$$\delta(m_1) = \{\{g_1, g_2\}, \{g_3, g_4\}\}$$

$$\delta(m_2) = \{\{g_2\}, \{g_1, g_3, g_4\}\}$$

$$\delta(m_3) = \{\{g_1\}, \{g_2, g_3, g_4\}\}$$

$$\delta(m_4) = \{\{g_1\}, \{g_2\}, \{g_3, g_4\}\}$$

$$\delta(m_5) = \{\{g_3\}, \{g_1, g_2, g_4\}\}$$

Definition

The meet and join of two partitions $d_1 = \{p_i\}$ and $d_2 = \{p_j\}$ are defined as:

$$d_1 \sqcap d_2 = \bigcup_{i,j} \{p_i \cap p_j\} \qquad d_1 \sqcup d_2 = \left(\bigcup_{p_i \cap p_j \neq \emptyset} \{p_i \cup p_j\} \right)^+$$

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$$d_1 \sqsubseteq d_2 \iff d_1 \sqcap d_2 = d_1$$

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Example

$$\delta(m_1) = \{\{g_1, g_2\}, \{g_3, g_4\}\}$$

$$\delta(m_2) = \{\{g_2\}, \{g_1, g_3, g_4\}\}$$

$$\delta(m_3) = \{\{g_1\}, \{g_2, g_3, g_4\}\}$$

$$\delta(m_4) = \{\{g_1\}, \{g_2\}, \{g_3, g_4\}\}$$

$$\delta(m_5) = \{\{g_3\}, \{g_1, g_2, g_4\}\}$$

$$\delta(m_1) \sqcap \delta(m_2) = \{\{g_1\}, \{g_2\}, \{g_3, g_4\}\}$$

$$\delta(m_1) \sqcup \delta(m_2) =$$

$$\{\{g_1, g_2\}, \{g_1, g_2, g_3, g_4\}, \{g_1, g_3, g_4\}\}^+$$

$$= \{\{g_1, g_2, g_3, g_4\}\}$$

Definition

A partition pattern structures for CC biclustering is determined by the triple $(M, (D, \sqsupseteq), \delta)$.

A pair (A, d) is then called a partition pattern concept (pp-concept) iff $A^{\square} = d$ and $d^{\square} = A$, where:

$$A^{\square} = \bigsqcap_{m \in A} \delta(m) \qquad A \subseteq M$$

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For any partition component $p \in d$, each pair (p, A) corresponds to a CC bicluster.

Partition Pattern Concept

Example

	m_1	m_2	m_3	m_4	m_5
g_1	1	5	3	4	7
g_2	1	1	4	2	7
g_3	2	5	4	5	3
g_4	2	5	4	5	7

Example

	m_1	m_2	m_3	m_4	m_5
g_1	1	5	3	4	7
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g_3	2	5	4	5	3
g_4	2	5	4	5	7

A pp-concept: $(\{m_1, m_2, m_3, m_4\}, \{\{g_1\}, \{g_2\}, \{g_3, g_4\}\})$.

Example

	m_1	m_2	m_3	m_4	m_5
g_1	1	5	3	4	7
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g_4	2	5	4	5	7

A pp-concept: $(\{m_1, m_2, m_3, m_4\}, \{\{g_1\}, \{g_2\}, \{g_3, g_4\}\})$.

CC biclusters:

- $(\{g_1\}, \{m_1, m_2, m_3, m_4\})$
- $(\{g_2\}, \{m_1, m_2, m_3, m_4\})$
- $(\{g_3, g_4\}, \{m_1, m_2, m_3, m_4\})$

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The rating of I (a set of items) according to V (a set of visitors)

	i_1	i_2	i_3	i_4	i_5
v_1	1	1	3	2	3
v_2	1	1	3	2	3
v_3	1	1	3	2	2
v_4	3	2	1	3	1
v_5	3	2	1	3	1
v_6	3	1	2	2	3
v_7	3	1	2	2	3
v_8	2	3	3	2	2
v_9	3	2	3	3	3
v_t	?	?	?	1	3

Recommendation

	i_1	i_2	i_3	i_4	i_5
v_1	1	1	3	2	3
v_2	1	1	3	2	3
v_3	1	1	3	2	2
v_4	3	2	1	3	1
v_5	3	2	1	3	1
v_6	3	1	2	2	3
v_7	3	1	2	2	3
v_8	2	3	3	2	2
v_9	3	2	3	3	3
v_t	?	?	?	1	3

Recommendation

	i_1	i_2	i_3	i_4	i_5
v_1	1	1	3	2	3
v_2	1	1	3	2	3
v_3	1	1	3	2	2
v_4	3	2	1	3	1
v_5	3	2	1	3	1
v_6	3	1	2	2	3
v_7	3	1	2	2	3
v_8	2	3	3	2	2
v_9	3	2	3	3	3
v_t	?	?	?	1	3

Recommendation for v_t : i_3

Recommendation

	i_1	i_2	i_3	i_4	i_5
v_1	1	1	3	2	3
v_2	1	1	3	2	3
v_3	1	1	3	2	2
v_4	3	2	1	3	1
v_5	3	2	1	3	1
v_6	3	1	2	2	3
v_7	3	1	2	2	3
v_8	2	3	3	2	2
v_9	3	2	3	3	3
v_t	?	?	?	1	3

Recommendation

	i_1	i_2	i_3	i_4	i_5
v_1	1	1	3	2	3
v_2	1	1	3	2	3
v_3	1	1	3	2	2
v_4	3	2	1	3	1
v_5	3	2	1	3	1
v_6	3	1	2	2	3
v_7	3	1	2	2	3
v_8	2	3	3	2	2
v_9	3	2	3	3	3
v_t	?	?	?	1	3

Recommendation for v_t : i_3

Recommendation

	i_1	i_2	i_3	i_4	i_5
v_1	1	1	3	2	3
v_2	1	1	3	2	3
v_3	1	1	3	2	2
v_4	3	2	1	3	1
v_5	3	2	1	3	1
v_6	3	1	2	2	3
v_7	3	1	2	2	3
v_8	2	3	3	2	2
v_9	3	2	3	3	3
v_t	?	?	?	1	3

Recommendation

	i_1	i_2	i_3	i_4	i_5
v_1	1	1	3	2	3
v_2	1	1	3	2	3
v_3	1	1	3	2	2
v_4	3	2	1	3	1
v_5	3	2	1	3	1
v_6	3	1	2	2	3
v_7	3	1	2	2	3
v_8	2	3	3	2	2
v_9	3	2	3	3	3
v_t	?	?	?	1	3

v_3 may dislike i_1 and like i_3 .

Recommendation

	i_1	i_2	i_3	i_4	i_5
v_1	1	1	3	2	3
v_2	1	1	3	2	3
v_3	1	1	3	2	2
v_4	3	2	1	3	1
v_5	3	2	1	3	1
v_6	3	1	2	2	3
v_7	3	1	2	2	3
v_8	2	3	3	2	2
v_9	3	2	3	3	3
v_t	?	?	?	1	3

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Recommendation for v_t : i_3

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A coherent-evolution-on-columns (CEC) bicluster

	m_1	m_2	m_3	m_4	m_5
g_1	1	2	3	4	5
g_2	4	2	1	?	3
g_3	2	3	4	1	1
g_4	5	4	2	3	1
g_5	2	1	5	4	3

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Partition

	m_1	m_2	m_3	m_4	m_5
g_1	1	2	3	4	5
g_2	4	2	1	?	3
g_3	2	3	4	1	1
g_4	5	4	2	3	1
g_5	2	1	5	4	3

$$p_{1,2} = (m_1, m_2)$$

Partition

	m_1	m_2	m_3	m_4	m_5
g_1	1	2	3	4	5
g_2	4	2	1	?	3
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$$P_{1,2} = (m_1, m_2)$$

$$\gamma : P \rightarrow D$$

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$$P_{1,2} = (m_1, m_2)$$

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$$\gamma(P_{1,2}) = \{\{g_1, g_3\}, \{g_2, g_4, g_5\}\}$$

	m_1	m_2	m_3	m_4	m_5
g_1	1	2	3	4	5
g_2	4	2	1	?	3
g_3	2	3	4	1	1
g_4	5	4	2	3	1
g_5	2	1	5	4	3

$$p_{1,4} = (m_1, m_4)$$

$$\gamma : P \rightarrow D$$

$$\gamma(p_{1,4}) = \{\{g_1, g_2, g_5\}, \{g_2, g_3, g_4\}\}$$

	m_1	m_2	m_3	m_4	m_5
g_1	1	2	3	4	5
g_2	4	2	1	?	3
g_3	2	3	4	1	1
g_4	5	4	2	3	1
g_5	2	1	5	4	3

$$P_{4,5} = (m_4, m_5)$$

$$\gamma : P \rightarrow D$$

$$\gamma(P_{4,5}) = \{\{g_1, g_2, g_3\}, \{g_2, g_3, g_4, g_5\}\}$$

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Example

$$\gamma(p_{1,2}) = \{\{g_1, g_3\}, \{g_2, g_4, g_5\}\}$$

$$\gamma(p_{1,4}) = \{\{g_1, g_2, g_5\}, \{g_2, g_3, g_4\}\}$$

$$\gamma(p_{1,2}) \sqcap \gamma(p_{1,4}) = \{\{g_1\}, \{g_3\}, \{g_2, g_5\}, \{g_2, g_4\}\}$$

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A pp-concept contains a CEC bicluster if there is a clique among the attributes in the pairs.

Partition Pattern Concept

Example

	m_1	m_2	m_3	m_4	m_5
g_1	1	2	3	4	5
g_2	4	2	1	?	3
g_3	2	3	4	1	1
g_4	5	4	2	3	1
g_5	2	1	5	4	3

Example

	m_1	m_2	m_3	m_4	m_5
g_1	1	2	3	4	5
g_2	4	2	1	?	3
g_3	2	3	4	1	1
g_4	5	4	2	3	1
g_5	2	1	5	4	3

A pp-concept: $\{\{p_{1,2}, p_{1,3}, p_{2,3}\}, \{\{g_1, g_3\}, \{g_5\}, \{g_2, g_4\}\}\}$

Example

	m_1	m_2	m_3	m_4	m_5
g_1	1	2	3	4	5
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A pp-concept: $\{\{p_{1,2}, p_{1,3}, p_{2,3}\}, \{\{g_1, g_3\}, \{g_5\}, \{g_2, g_4\}\}\}$

Biclusters:

- $(\{g_1, g_3\}, \{m_1, m_2, m_3\})$
- $(\{g_2, g_4\}, \{m_1, m_2, m_3\})$
- $(\{g_5\}, \{m_1, m_2, m_3\})$

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Order of visit of 7 items

	i_1	i_2	i_3	i_4	i_5	i_6	i_7
v_1	1	2	3	4	5	6	7
v_2	2	4	5	3	7	1	6
v_3	4	2	1	5	6	3	7
v_4	7	3	1	4	2	6	5
v_t	?	?	1	2	?	?	?

Recommendation

	i_1	i_2	i_3	i_4	i_5	i_6	i_7
v_1	1	2	3	4	5	6	7
v_2	2	4	5	3	7	1	6
v_3	4	2	1	5	6	3	7
v_4	7	3	1	4	2	6	5
v_t	?	?	1	2	?	?	?

Recommendation for v_t : i_5 .

Order of interest of 5 items

	i_1	i_2	i_3	i_4	i_5
v_1	1	2	3	4	5
v_2	3	?	4	2	1
v_3	2	4	3	?	1
v_t	1	1	2	?	?

Recommendation

	i_1	i_2	i_3	i_4	i_5
v_1	1	2	3	4	5
v_2	3	?	4	2	1
v_3	2	4	3	?	1
v_t	1	1	2	?	?

Recommendation

	i_1	i_2	i_3	i_4	i_5
v_1	1	2	3	4	5
v_2	3	?	4	2	1
v_3	2	4	3	?	1
v_t	1	1	2	?	?

Recommendation for v_t : i_5 .

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- We have explored the approaches to build collaborative recommendation strategies for visitors in a museum.

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- Comparison of CEC biclustering with sequential pattern mining?
- Filtering the biclusters based on “score”?

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- We also presented a technique for mining CC and CEC biclusters based on FCA using pattern structures.
- Comparison of CEC biclustering with sequential pattern mining?
- Filtering the biclusters based on “score”?
- Implementation of CEC biclustering?

Thank you